One-fluid description of turbulently flowing suspensions

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We suggested a *one-fluid* model of a turbulent dilute suspension which accounts for the "two-way" fluid-particle interactions by k-dependent effective density of suspension and additional damping term in the Navier-Stokes equation. We presented analytical description of the particle modification of turbulence including scale invariant suppression of a small k part of turbulent spectrum (independent of the particle response time) and possible enhancement of large k region [up to the factor $(1+\phi)^{2/3}$]. Our results are in agreement with qualitative picture of isotropic homogeneous turbulence of dilute suspensions previously observed in laboratory and numerical experiments.

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Introduction. The interaction of solid particles or liquid droplets with turbulence in gases controls the performance of various engineering devices like the combustion of pulverized coal and liquid sprays and cyclone separations. This interaction plays an important role in many areas of environmental science and physics of the atmosphere. Dust storms, rain triggering, dusting and spraying for agricultural or forestry purposes, preparation and processing of aerosols are typical examples. For a review of turbulent flows with particles and droplets see, e.g. [1].

In dilute suspensions with small volume fractions of particles, $C_{\rm p}$, the particle-particle interactions are negligible. Nevertheless, for $\rho_{\rm p}/\rho_{\rm f} \gg 1$ (the ratio of the solid particle material and the gas densities), the mass loading $\phi = C_{\rm p}\rho_{\rm p}/\rho_{\rm f}$ may exceed unity and kinetic energies of the particles and the carrier gas may be compatible; hence the "two-way coupling": effect of fluid on particles and vice versa must be accounted for. Current understanding of turbulence in dilute suspensions is still at its infancy due to the highly nonlinear nature of physically relevant interactions and a wide spectrum of governing parameters (the particle size a vs. L and η, the outer and inner scales of turbulence, the particle response time $\tau_{\rm p}$ vs. $\gamma_{_L}$ and γ_η, turnover frequencies of L- and η- scale eddies).

Analytical study of the problem is mainly based upon a *two-fluid* model wherein both the carrying fluid and particle phases are treated as interpenetrating continua [1, 2, 3]. This model deals with non-interacting solid spherical particles which are small enough such that: (i) one can neglect the effect of preferential concentration and assume homogeneity of the particle space distribution; (ii) the Stokes viscous drag law for particle acceleration, $d\boldsymbol{u}_{\rm p}/dt = [\boldsymbol{u}_{\rm f} - \boldsymbol{u}_{\rm p}]/\tau_{\rm p}$, is valid ($\boldsymbol{u}_{\rm f}$ is the fluid velocity). Unfortunately, statistical description of two-fluid turbulence by closure procedures requires a set of additional questionable simplifications due to the lack of understanding of the relevant physics of the particle-fluid interactions. This made closures of the two-fluid model highly qualitative at best [3, 4, 5].

We think that the basic physics of the problem may be described by a simpler *one-fluid model* of turbulent dilute suspensions, which requires standard closures of one-phase turbulence. The present Letter suggests such a model and, as a first step, uses a properly modified simple closure, based on the Kolmogorov-Richardson cascade picture of turbulence. The resulting non-linear differential equations for the energy budget were solved analytically with required accuracy. This provides an economical and internally consistent analytical description of the turbulence modification by particles including the dependencies of suppression or enhancement of the turbulent activity on the three governing parameters: $(\tau_{\rm p}\gamma_{_L})$, ϕ and scale of eddies. These effects were previously observed in numerous experimental and numerical studies [1, 2, 3, 4, 5, 6], but they still await analytical description. Our analytical findings are in a qualitative agreements with the observations. We believe that the one-fluid model, together with more elaborated closures of one-phase turbulence [7], offers an insightful and constructive look at basic physics of even more complex particle-laden turbulent flows.

1. One-fluid model of turbulent suspensions reads:

$$
\begin{aligned}
&\rho_{\rm eff}(k)\Big[\frac{\partial}{\partial t} + \gamma_{\rm p}(k)\Big]\boldsymbol{u}(t,\boldsymbol{k}) + \mu k^2 \boldsymbol{u}(t,\boldsymbol{k}) \qquad (1)\\
&+(2\pi)^{-3}\int {\rm d}^3k_1\,{\rm d}^3k_2 \Gamma^{\alpha\beta\gamma}_{\boldsymbol{k}\,\boldsymbol{k}_1\boldsymbol{k}_2} u^*_\beta(t,\boldsymbol{k}_1) u^*_\gamma(t,\boldsymbol{k}_2) = 0\ .
\end{aligned}
$$

Here $\boldsymbol{u}(t,\boldsymbol{k})$ is the incompressible velocity field of the carrier fluid in the $\boldsymbol{k}$ representation and μ is the dynamical viscosity. This equation differs from the Navier-Stokes (NS) equation in the three aspects:

a. The carrier fluid density $\rho_{\rm f}$ is replaced by $\rho_{\rm eff}(k)$, a k-dependent effective density of the suspension for turbulent fluctuations of scale $1/k$ [referred to as k-eddies];

b. The fluid-particle friction is described by a damping frequency $\gamma_{\rm p}(k)$.

c. The interaction amplitude of the NS equation, $\gamma^{\alpha\beta\gamma}_{\boldsymbol{k}\,\boldsymbol{k}_1\boldsymbol{k}_2} = \rho_{\rm f}\big[P^{\alpha\beta}(\boldsymbol{k})\,k^\gamma + P^{\alpha\gamma}(\boldsymbol{k})\,k^\beta\big]\delta(\boldsymbol{k}+\boldsymbol{k}_1+\boldsymbol{k}_2)/2$, $P^{\alpha\beta}(\boldsymbol{k}) = \delta_{\alpha\beta} - k^\alpha k^\beta/k^2$ is replaced by

$$
\Gamma^{\alpha\beta\gamma}_{\boldsymbol{k}\,\boldsymbol{k}_1\boldsymbol{k}_2} = \rho_{\rm eff}\big[2\,k_1k_2k_3/(k_1^2+k_2^2+k_3^2)\big]\gamma^{\alpha\beta\gamma}_{\boldsymbol{k}\,\boldsymbol{k}_1\boldsymbol{k}_2}/\rho_{\rm f}\ . \quad (2)
$$

An underlying physics of these aspects is quite simple:

a. For small k the turnover frequency $\gamma(k)$ of k-eddies is small in the sense $\gamma(k)\tau_{\rm p} \ll 1$. Therefore in this region of k the particle velocity is very close to that of the carrier fluid and we can describe the suspension as *one fluid* with effective density $\rho_{\rm eff}(k)$ which is close to the density of suspension, $\rho_{\rm s} = \rho_{\rm f} + C_{\rm p}\,\rho_{\rm p}$. In contrary, for large k, when $\gamma(k)\tau_{\rm p} \gg 1$, the particles cannot follow these very fast motions and may be considered at rest. Thus the particles do not contribute to the effective density and $\rho_{\rm eff}(k) \to \rho_{\rm f}$. In general case $\rho_{\rm eff}(k)$ may be given by

$$\rho_{\rm eff}(k) = \rho_{\rm f}\big[1+\phi f_{\rm com}(k)\big]\,, \quad \phi = C_{\rm p}\,\rho_{\rm p}/\rho_{\rm f}\ . \tag{3}$$

Here a statistical ensemble of all particles, partially involved in the motion of k-eddies, is replaced by two subensembles of "fully comoving" fraction $f_{\rm com}(k)$ of particles and "fully resting" fraction $f_{\rm rest}(k) = 1 - f_{\rm com}(k)$ of particles, which does not contribute to $\rho_{\rm eff}(k)$.

b. The particles at rest cause fluid-particle friction. According to Newton's third law, the damping frequency of suspension $\gamma_{\rm p}(k)$ may be related to the particle response time, $\tau_{\rm p}$, via the ratio of total mass of particles $M_{\rm p}$ to the total effective mass of the suspension $M_{\rm eff}(k)$:

$$\gamma_{\rm p}(k) = \frac{M_{\rm p}}{\tau_{\rm p}\, M_{\rm eff}(k)} = \frac{C_{\rm p}\,\rho_{\rm p} f_{\rm rest}(k)}{\tau_{\rm p}\,\rho_{\rm eff}(k)} = \frac{\phi\,\rho_{\rm f}\, f_{\rm rest}(k)}{\tau_{\rm p}\,\rho_{\rm eff}(k)}\ . \tag{4}$$

As we mentioned, the portions $f_{\rm com}(k)$ and $f_{\rm rest}(k)$ depend on $\tau_{\rm p}\gamma(k)$. Clearly, for $\tau_{\rm p}\gamma(k) \ll 1$, $f_{\rm rest}(k)$ has the same smallness: $f_{\rm rest}(k) \sim \tau_{\rm p}\gamma(k)$. In the opposite case when $1/\tau_{\rm p}\gamma(k)$ is small, $f_{\rm com}\,(k)$ has corresponding smallness: $f_{\rm com}(k) \sim 1/\tau_{\rm p}\gamma(k)$. For $\tau_{\rm p}\gamma(k) = 1$ one expects $f_{\rm rest}(k) \simeq f_{\rm com}(k) \simeq \frac{1}{2}$. As a simple model of such a function we adopt

$$f_{\rm rest}(k) \ = \ 1 - f_{\rm com}(k) = \tau_{\rm p}\gamma(k)/[1+\tau_{\rm p}\gamma(k)]\,, \tag{5}$$

which also follows from more elaborated analysis [7]. With Eq. (5) we rewrite Eqs. (3) and (4) as follows:

$$\begin{aligned} \rho_{\rm eff}(k) \ &= \ \rho_{\rm f}\big\{1+\phi/[1+\tau_{\rm p}\gamma(k)]\big\}\,, & (6)\\ \gamma_{\rm p}(k) \ &= \ \phi\,\gamma(k)/[1+\phi+\tau_{\rm p}\gamma(k)]\ . & (7)\end{aligned}$$

c. It may be shown that interaction amplitude $\Gamma^{\alpha\beta\gamma}_{\boldsymbol{k}\,\boldsymbol{k}_1\boldsymbol{k}_2}$ is *Galilean invariant* and *conserves the total kinetic energy* of suspension (i.e. of the carrier fluid and fully comoving part of the particles) if one neglects the damping terms μk^2 and $\gamma_{\rm p}(k)$ in Eq. (1). Notice, that the detailed form of $\Gamma^{\alpha\beta\gamma}_{\boldsymbol{k}\,\boldsymbol{k}_1\boldsymbol{k}_2}$ is not important for this discussion, only the conservation of the energy is essential.

The basic equations of our model, (1), (2), (6) and (7) are *self-consistent equations for turbulent velocity of the carrier fluid* in which the coefficients depend on the eddy-turnover frequency $\gamma(k)$ which, in its turn, depends on a stochastic solution of the same equations.

2. Budget of the kinetic energy in suspensions. The definition of one-dimensional density of kinetic energy in a *single phase flow* $E(t,k)$ reads

$$E(t,k) \ = \ \rho_{\rm f} k^2 F_2(t,k)/2\pi\ . \tag{8}$$

Here $F_2(t,k)$ is the simultaneous pair velocity correlation for isotropic turbulence. Corresponding definition of one-dimensional density of kinetic energy *for suspensions*, $\mathcal{E}(k)$, has to account for the k-dependent density:

$$\mathcal{E}(t,k) \ = \ \rho_{\rm eff}(k) k^2 F_2(t,k)/2\pi\ . \tag{9}$$

Multiplying Eq. (1) by $\boldsymbol{u}(t,\boldsymbol{k}')$ and averaging, one gets the equation for the energy budget in the inertial interval:

$$\mathcal{E}(t,k)/2\,\partial t + \gamma_{\rm p}(k)\mathcal{E}(t,k) + {\rm d}\varepsilon(k)/{\rm d}k = 0\ . \tag{10}$$

Here $\varepsilon(k)$ is *one-dimensional energy flux* in the k-space and we neglected the energy input in the outer scale L and the viscous damping, μk^2, which becomes essential near the viscous microscale η.

In the Richardson-Kolmogorov picture of turbulence the only relevant parameters in the inertial interval are k, $\rho_{\rm f}$ and $\varepsilon(k)$. Similarly, in our model (1) these parameters are k, $\rho_{\rm eff}(k)$ and $\varepsilon(k)$. Other functions in the problem may be related to them by the dimensional reasoning:

$$\begin{aligned} \mathcal{E}(k) \ &= \ C_1\left[\varepsilon(k)^2\rho_{\rm eff}(k)\right]^{1/3} k^{-5/3}\,, & (11)\\ \gamma(k) \ &= \ C_2\left[\varepsilon(k)/\rho_{\rm eff}(k)\right]^{1/3} k^{2/3}\,, & (12)\end{aligned}$$

where $C_1 \sim C_2 \sim 1$ are dimensionless constants for particle free case. Hereafter we omit the explicit reference to the time dependence. Substituting Eqs. (6), (7), (11) and (12) into Eq. (10) yields in the stationary case

$$\frac{{\rm d}\varepsilon(k)}{{\rm d}k} + \frac{\varepsilon(k)}{k}\,\frac{C\,\phi}{1+\phi+\gamma(k)\tau_{\rm p}} = 0\,, \quad C = C_1\,C_2\ . \tag{13}$$

To find the iterative solution of Eq. (13) we denote as $\varepsilon_n(k)$, $\gamma_n(k)$ and $\rho_{{\rm eff},n}(k)$ corresponding functions at nth iteration step; take for "zeroth step" their values at $k_{\rm outer} = 1/L$: $\varepsilon_0(k) = \varepsilon(1/L) \equiv \varepsilon_{_L}$, $\gamma_0(k) = \gamma(1/L) \equiv \gamma_{_L}$, $\rho_{{\rm eff},0}(k) = \rho_{\rm eff}(1/L) \equiv \rho_{_L}$; define dimensionless functions of $\kappa \equiv kL$: $\varepsilon_n(k) = \varepsilon_{_L} f_n(\kappa)$, $\gamma_n(k) = \gamma_{_L} g_n(\kappa)$, $\rho_{{\rm eff},n}(k) = \rho_{\rm f}\, r_n(\kappa)$ and iterate the equations

$$\begin{aligned} f_n(\kappa) \ &= \ \exp\left\{\int_1^\kappa \frac{-\Delta\,{\rm d}x}{x\,[1+\delta g_{n-1}(x)]}\right\}\,, \quad \delta = \frac{\tau_{\rm p}\gamma_{_L}}{1+\phi}\,,\\ g_n(\kappa) \ &= \ \Big[\frac{\kappa^2 f_n(\kappa) r_{n-1}(1)}{r_{n-1}(\kappa)}\Big]^{1/3}\,, \qquad \Delta = \frac{C\,\phi}{1+\phi}\,,\\ r_n(\kappa) \ &= \ 1+\phi/[1+\tau_{\rm p}\gamma_{_L}\, g_n(\kappa)]\,, & (14)\end{aligned}$$

which coincide with (13), (12), (7) after ignoring sub-

TABLE I: Parameters ϕ and $\tau_{\rm p}$ for numerical solution of Eqs. (11) – (13) with $C_1 = C_2 = \frac{1}{2}$, and computed values of δ, crossover scale, k_*, and normalized rates of dissipation: by the iteration procedure, $f_{2\infty}$, $f_{3\infty}$ and numerically, f_∞.

ϕ	1.0					0.75	0.5	0.25
$\tau_{\rm p}$	1.0	0.5	0.2	0.1	0.02	0.5		
δ	0.21	0.10	0.04	0.02	0.004	0.12	0.15	0.19
$\tau_{\rm p}\gamma_L$	0.42	0.20	0.08	0.04	0.008	0.21	0.22	0.24
k_*	3.4	9.5	39	117	1530	9.2	8.8	8.4
$f_{2,\infty}$	0.704	0.621	0.522	0.455	0.331	0.686	0.766	0.868
$f_{3,\infty}$	0.721	0.639	0.537	0.469	0.341	0.699	0.774	0.871
f_∞	0.723	0.642	0.541	0.474	0.352	0.701	0.775	0.871

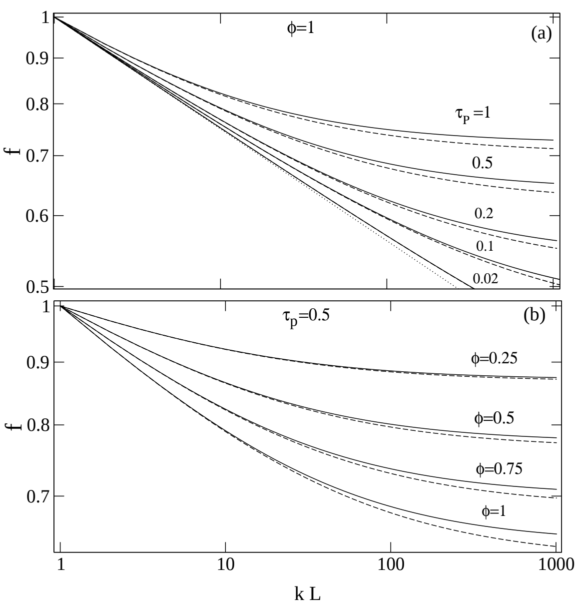

FIG. 1: Log-Log plots of solutions of Eqs. (11) – (13) for parameters in Tab. I. Solid lines: "exact" numerical solutions $f(kL) = \varepsilon(k)/\varepsilon_L$, dashed lines: $f_2(kL) = \varepsilon_2(k)/\varepsilon_L$, Eq. (16).

script "$_n$". First two iterations may be done explicitly:

$$\begin{aligned}
f_1(\kappa) &= \frac{\varepsilon_1(k)}{\varepsilon_L} = \kappa^{-\Delta}\,, \quad g_1(\kappa) = \frac{\gamma_1(k)}{\gamma_L} = \kappa^{(2-\Delta)/3}\,, \\
r_1(\kappa) &= \frac{\rho_{\rm eff,1}(k)}{\rho_{\rm f}} = 1 + \frac{\phi}{1+\tau_{\rm p}\gamma_L\kappa^{(2-\Delta)/3}}\,; \qquad (15) \\
f_2(\kappa) &= \frac{\varepsilon_2(k)}{\varepsilon_L} = \Big[\frac{\delta + \kappa^{(\Delta-2)/3}}{\delta+1}\Big]^{3\Delta/(2-\Delta)}\,, \qquad (16) \\
g_2(\kappa) &= \gamma_2(k)/\gamma_L = \kappa^{2/3} f_2^{1/3}(\kappa) r_1^{-1/3}(\kappa)\,, \\
r_2(\kappa) &= \rho_{\rm eff,2}(k)/\rho_{\rm f} = 1 + \phi/[1+\tau_{\rm p}\gamma_L\, g_2(\kappa)]\,.
\end{aligned}$$

The third iteration requires simple numerical integration:

$$\varepsilon_3(k) = \varepsilon_L \exp\Big\{-\Delta\int_1^{kL} {\rm d}x/x\,[1+\delta g_2(x)]\Big\}\,. \quad (17)$$

Figure 1 displays "exact" numerical solutions of $\varepsilon(k)/\varepsilon_L$ (for sets of ϕ and $\tau_{\rm p}$ listed in Tab. I) in comparison with corresponding plots of Eq. (16) for $f_2(\kappa)$. In the first decade of the inertial interval ($kL \le 10$) discrepancies between the numerics and the second iteration are quite small. They gradually increase toward large k, remaining smaller than $(1-2)\%$. Values of $f_\infty \equiv f(\infty)$, $f_{2,\infty} = [\delta/(1+\delta)]^{3\Delta/(2-\Delta)}$ and $f_{3,\infty}$ following from Eq. (17) are given in Tab. I. It is clear that almost always one may use simple analytical solution of the second iterations, (16). The results (17) of the third iteration may be used to control accuracy.

3. Turbulence modification by particles. In the particle-free turbulence the rate of energy input ε_L is equal to the energy flux in the inertial interval $\varepsilon(k)$ and to the rate of energy dissipation ε_∞. In turbulent suspensions due to fluid-particle friction $\varepsilon(k)$ is no longer constant and decreases toward large k. Therefore $\varepsilon_L > \varepsilon(k) > \varepsilon_\infty$. Eq. (15) and Fig. 1a show that in the region $k \ll k_*$ $[\tau_{\rm p} g(k_*) = 1]$ the flux decays toward small scale: $\varepsilon(k) \approx \varepsilon_L (kL)^{-\Delta}$ with Δ, Eq. (14), independent of $\tau_{\rm p}$, while for $k \gg k_*$ the flux approaches plato, similar to the particle-free case. At first glance these two facts are unexpected: there is an essential suppression of the large scale eddies in spite of the fact that particles are almost swept by them and therefore the fluid-particle damping $\gamma_{\rm p}(k)$ is small. On the contrary, the small scale eddies are almost not effected by particles which are not involved in their motion and thus $\gamma_{\rm p}(k)$ reaches the maximum value $\phi/\tau_{\rm p}$. To explain consider the dimensionless rate of the flux decrement, $[-d\ln\varepsilon(k)/d\ln k]$, which is $\propto \gamma_{\rm p}(k)$. The only available frequency to make $\gamma_{\rm p}(k)$ dimensionless is $\gamma(k)$. Therefore $-d\ln\varepsilon(k)/d\ln k \sim \Gamma(k) \equiv \gamma_{\rm p}(k)/\gamma(k)$, in agreement with Eq. (13). As one sees from Eq. (7)

$$\Gamma(k) = \phi/[1+\phi+\tau_{\rm p}\gamma(k)] \qquad (18)$$

and for $\tau_{\rm p}\gamma(k) \ll 1$ the ratio $\Gamma(k)$ becomes $\tau_{\rm p}$ independent constant. This explains both facts: ***a*** - why the suppression of small k eddies is $\tau_{\rm p}$ independent and ***b*** - why this suppression is scale invariant. Note that the weak sensitivity of turbulent spectra to $\tau_{\rm p}$ (fact ***a***) was previously observed in numerous simulations of turbulence in dilute suspensions but, to the best of our knowledge, was not well understood, see, e.g. [3]. Equation (18) shows that $\Gamma(k) \to 0$ for $\tau_{\rm p}\gamma(k) \gg 1$. Therefore in this region of k particles cannot modify the turbulence and indeed $\varepsilon(k)$ must approach a constant value, ε_∞.

For brief comparison of the prediction ***b*** with direct numerical simulation by Boivin, Simonin and Squires [3]) we reploted in Fig. 2 their Fig. 5b for kinetic energy spectra $E(k)$ of suspensions in Log-Log coordinates (lower lines). Our first iteration (15) predicts $E(k) \propto k^{-5/3-2\Delta/3}$ with $\Delta = C\phi/(1+\phi)$. Therefore we defined "compensated" spectra as $E_{\rm c}(k,\phi) = E(k)(kL)^{5/3+2\Delta/3}$ and plotted them in Fig. 2 (with $C = 3.8$). The region 0.4 <Log $k < 1$ where for $\phi = 0$, $E_{\rm c}(k) = E(k)(kL)^{5/3}$, upper solid line, is approximately constant may be considered as inertial interval. As we expected, in this interval all lines

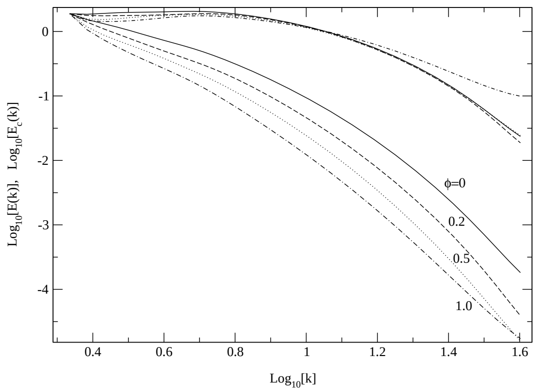

FIG. 2: Log-Log plots of turbulent kinetic energy spectrum $E(k)$ taken from [3] for $\phi = 0, 0.2, 0.5$ and 1 (lower lines) and "compensated" spectra $E_c(k)$ for the same ϕ (upper lines).

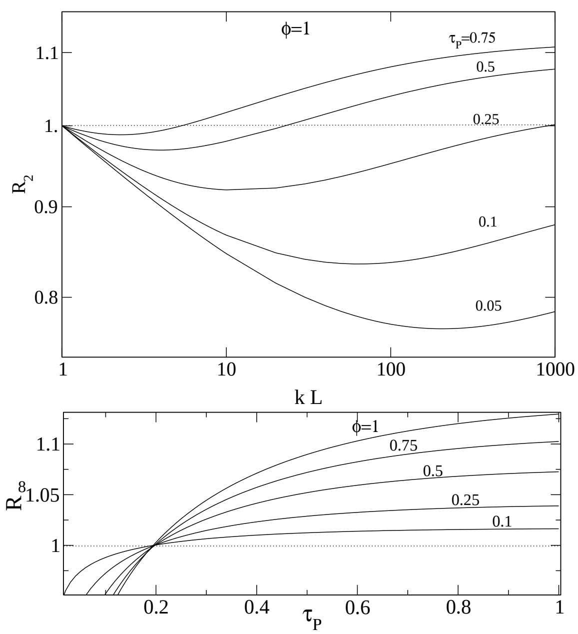

FIG. 3: Upper panel: Log-log plots of functions $R_2(k) = [f_2(\kappa)r_2(1)/r_2(\kappa)]^{2/3}$, Eqs. (16), (19), for $\phi = 1$ and values of τ_p denoting corresponding lines. Lower panel: Plots of $R_{2,\infty}$, Eq. (20), vs. τ_p. Values of ϕ label corresponding lines.

for different ϕ are about to collapse. Some scattering of the lines is related with k-dependence of $\rho_\mathrm{eff}(k)$ and finite value of τ_p which is neglected in the first iteration. More detailed comparison of our second and next order iterations will be done elsewhere [7].

Consider now a possible enhancement of the density of kinetic energy of the carrier fluid $E(k)$. According to Eqs. (8), (9) and (11) $E(k) = C_1\rho_\mathrm{f}\left[\varepsilon(k)/\rho_\mathrm{eff}(k)\right]^{2/3}k^{-5/3}$. Introduce the dimensionless ratio

$$R(k) \equiv \frac{E(k)/E(1/L)}{E_0(k)/E_0(1/L)} = \left[\frac{\varepsilon(k)\rho_{_L}}{\varepsilon_{_L}\rho_\mathrm{eff}(k)}\right]^{2/3} . \quad (19)$$

Here $E_0(k) = C_1\varepsilon^{2/3}\rho_\mathrm{f}^{1/3}k^{-5/3}$ is the density of turbulent kinetic energy in the particle-free case. This ratio is larger (smaller) than unity in the case of enhancement (suppression) of the turbulent energy by particles. To understand this behavior consider three distinct regions of scales defined by the crossover scale k_* (see Table I):

a. *Region of small scales*, $L^{-1} < k < k_*$, where $\varepsilon(k)$ is decreasing function of k. Function $\rho_\mathrm{eff}(k)$ in the denominator of Eq. (19) is almost constant $\rho_\mathrm{f}(1+\phi)$.

b. *Region of transient scales*, $k \sim k_*$, where $\varepsilon(k)$ still decreases, and so does $\rho_\mathrm{eff}(k)$ gradually reducing to ρ_f.

c. *Region of large scales*, $k > k_* > \eta^{-1}$, where $\varepsilon(k)$ approaches plato, while $\rho_\mathrm{eff}(k)$ is again constant, ρ_f. It is clear that behavior of $R(k)$ will depend on k_*L. For $k_*L \gg 1$ (small enough τ_p) the ratio $R(k)$ has enough room in the region ***a*** to strongly decay [as $(kL)^{-2\Delta/3}$] due to decay of $\varepsilon(k)$. In the region ***b*** it may increase [due to decrease of $\rho_\mathrm{eff}(k)$] but not more than by factor of $(1+\phi)^{2/3}$. Therefore for small enough τ_p the ratio $R(k) < 1$ everywhere, see, e.g., in Fig. 3a plots for $\tau_p = 0.05$,($k_*L \approx 350$) and $\tau_p = 0.1$ ($k_*L = 117$).

There is an essential enhancement of the kinetic energy for larger τ_p (smaller k_*L), when the small scale region ***a*** is not pronounced (plots for $\tau_\mathrm{p} = 0.5$ ($k_*L = 9.5$) and $\tau_\mathrm{p} = 0.75$ ($k_*L = 5.2$) in Fig. 3a). In this case the growth of $R(k)$ in the transient region ***b*** is stronger than the decay in the region ***a***.

To find values of parameters for which the enhancement is possible (at least for $k \to \infty$) consider the maximum value of $R_\infty = R(\infty)$. In the second iteration

$$R_{2,\infty} = \left(\frac{\delta}{1+\delta}\right)^{2\Delta/(2-\Delta)}\left(1 + \frac{\delta\phi}{1+\delta+\phi}\right)^{2/3} . \quad (20)$$

Fig. 3b displays plots $R_{2,\infty}$ vs. τ_p for different ϕ. The enhancement is possible if $\delta = \tau_\mathrm{p}\gamma_{_L}/(1+\phi)$ exceeds a critical value δ_cr which is independent of ϕ. Parameter ϕ governs the value of the enhancement. The maximal possible enhancement (for large kL and δ) is $(1+\phi)^{2/3}$.

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